TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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Abstract of paper entitled "Über Aneroide," by E. Warburg and W. Heuse, Physikalisch-Technischen Reichsanstalt, Zs. f. Instrumentenkunde 39:41-55, 1919.

Prepared by
M. D. Hersey, Massachusetts Institute of Technology.

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Following an historical review of aneroid development, in which the work of Chree is emphasized, the authors state:

"It would therefore seem well worth while to undertake, through some suitable improvement of the aneroid, to diminish the width of the hysteresis loop sufficiently to make the errors resulting from it practically negligible."

To discover the origin of the loops, experiments were made similar in purpose to the "vivisection experiments," so-called, which were made at the Bureau of Standards and referred to in the Physical Review, 6:75-77, 1915, article entitled, "Aneroid Barometers."

Concluding that the trouble is not in the steel spring, but rather in the diaphragm, a theoretical study is made of the equilibrium of forces acting on the vacuum box. After setting up the fundamental equation of equilibrium

$$T = T_0 + f\ z \qquad (1)$$

in which T denotes the tension pulling upward on the vacuum box, T_o its initial value when the downward deflection of the box is $z = o$ and when the air pressure p has its standard value p_o; and in which f is a stiffness constant for the spring - the authors proceed to elaborate the terms of this equation by reference to the theory of elasticity.

Two cases are considered, - first, that of a thin, flat diaphragm undergoing a deflection which is quite small compared to its thickness; second, the opposite extreme case when the deflection of the diaphragm is quite large compared to its thickness.

The first special case enables us to consider that only shearing stresses normal to the diaphragm exist, radial tensions being neglected. The other extreme case makes just the opposite assumption, namely, that the only stresses which need to be considered are tensions parallel to the surface of the diaphragm. After carrying through the theory of both cases as far as possible, the authors abandon the equations so deduced in favor of empirical observations. Nevertheless, some tentative conclusions are reached which are interesting.

Thus, for the flat diaphragm undergoing only an infinitesimal deflection, the following equations are derived from the theory developed by Foeppl, Vorlesungen uber technische Mechanik, 3:255, 1897; 5:110, 1907:

$$\left.\begin{aligned}
s^2 &= s_o^2 - \frac{8\alpha z}{p\rho^2} \\
T &= \pi p s_o^2 - \frac{p\pi\alpha z}{\rho^2} \\
s_o^2 &= \frac{\lambda^4}{\rho^2} \\
\lambda^4 &= \frac{1}{4}(r_2^4 - r_1^4) - r_1^2 r_2^2 \log\frac{r_2}{r_1} \\
\rho^2 &= r_2^2 - r_1^2 - \frac{4 r_1^2 r_2^2}{r_2^2 - r_1^2}\left(\log\frac{r^2}{r^1}\right)^2 \\
\alpha &= h^3 E \frac{m^2}{6(m^2-1)}
\end{aligned}\right\} \qquad (2)$$

In these equations, to begin with, s denotes the radius of a disk such that the force acting on it due to the pressure p will just equal the actual tension T, and s_o is the value which it has for zero deflection. The symbols α, ρ and λ employed in the first three equations are defined by the last three equations. (In the original text, P is employed for the tension; and the letter a, in German type, appears instead of our α). Further, h = thickness of diaphragm, E = modulus of elasticity and $1/m$ = cross-section contraction/longitudinal strain.

The authors explain why s_o is approximately equal to $(r_1 + r_2)/2$ where r_1 and r_2 are respectively the inner and outer radii of the annular diaphragm.

An original feature of the authors' contribution now appears in their statement that when the deflection is zero, the distribution of load between the rim and the central solid disk is independent of the elastic constants of the diaphragm, and the performance of the instrument momentarily independent, therefore, of elastic after-effect when passing through the position $z = o$. Conversely, if z is different from o, then, as a result of the term $\frac{8 \pi \alpha z}{\rho^2}$ which now enters, T and with it the distribution of air pressure between the rim and the plate will depend on the elastic reaction of the diaphragm. If, for example, the pressure is changed from p_o to a higher value p, the plate deflects inward, z takes on a positive value, and the elastic reaction of the diaphragm diminishes the proportion of the air pressure which falls on the plate. If any reaction diminishes because of elastic after-effect, then the plate will again be more heavily loaded and will continue its inwardly directed motion.

That part of the elastic reaction of the diaphragm which depends on z is, according to equation (2) independent of the pressure p, remaining, therefore, unaltered in magnitude for zero pressure.

If one substitutes into equation (1) the value of T obtained from equation (2) and takes account of the fact that, according to the equations (2)

$$T_o = \pi p_o s_o^2 \qquad (2a)$$

one finds for the sensitivity of aneroids with very small pressure changes

$$\frac{z}{p - p_o} = \frac{\pi s_o^2}{f + \frac{8 \pi \alpha}{\rho^2}} \qquad (3)$$

The aneroid diaphragms used were made of .02 cm. thick German silver for which, with $1/m = .37$, $E = 11 \times 10^5$ kg/cm², $\alpha = 1.7$.

From equation (3) it is evident that the influence of elastic after-effect on the indication of an aneroid in the contemplated case diminishes with diminishing thickness of the diaphragm, and with increasing spring constant f, this last insofar as the after-effect of the diaphragm is greater than that of the spring.

In the second extreme case, the authors neglect the bending stresses and stresses normal to the diaphragm, and confine their analysis to the tensile stresses in the neutral surface. Consider the radial distance r_o at which a tangent plane touches the surface of a diaphragm which is simultaneously deflected down by pressure p and up by tension T. At the point of tangency the tensile stresses have no vertical component; therefore

$$T = \pi p r_o^2 \qquad (4)$$

Thus the distribution of load between the outer ring and the central solid disk depends only on the value of this radius r_o.

For the case of zero deflection, supposing the diaphragm initially flat and free from tensile stresses, when $p = o$ and $T = \delta$, a solution is quoted from Foeppl, (Op. cit. 5:144, 1907) according to which the deflection ζ as a function of the variable radius r is given by the equation

$$\zeta = \sqrt[3]{\frac{p}{h\,E}}\;\Phi\;(r_1,\,r_2,\,r) \tag{5}$$

and therefore the distribution of load is here also independent of the nature of the diaphragm and consequently the after-effect has no influence on the displacement whenever the latter happens to be zero. But the cases when the displacement is not zero it has not been possible to treat theoretically.

The difficulties of treating corrugated diaphragms are then alluded to, and experiments described in which the diaphragms were loaded with weights T which were varied over a cycle for different constant values of the air pressure p.

The results could be represented by the formula

$$T = ap - Qz + Rz^2$$

where

$$Q = b + cp + dp^2 \tag{6}$$

$$R = \beta + \gamma p + \delta p^2$$

In most cases $R = 0$ so that

$$T = ap - (b + cp + dp^2)\;z \tag{6a}$$

As a first approximation it is shown that s_o is again approximately equal to the average of the two radii if one replaces the empirical constant a by the term $\pi\, s_o^{\,2}$. Hence, approximately the coefficient a is independent of the nature of the diaphragm so that here again the distribution of the load between the central disk and the rim can not be influenced by after-effect whenever the displacement is zero.

The coefficient b is found to be about forty times as large in the experiment as it would be from the first of the theoretical deductions; moreover, it is found experimentally that b is approximately proportional to the thickness of the diaphragm, whereas, according to the theory given, it should vary with the cube of the thickness. This seeming discrepancy is sufficiently explained by the fact that the observations dealt with large deflections, such as were not contemplated at all in the theory quoted; moreover, the observations were made on corrugated diaphragms.

It is concluded that the coefficients b, c and d, particularly the first, depend on the properties of the diaphragm metal and serve to explain the hysteresis loops.

Upon substituting the value of T from equation (6) into equation (1), and taking the approximate condition $R = 0$, there results the following approximate expression for the sensitivity of an aneroid in terms of the coefficients a, b, c, d and f. The authors consider that this expression warrants special attention because of its simplicity, and because it is reliable to 5% in the cases which they have investigated:

$$\frac{z}{p - p_0} = \frac{a}{f + Q} = \frac{a}{b_t\, f + cp + dp^2} \qquad (7b)$$

Dropping the theoretical discussion, the authors announce the conclusion that the means available to diminish the effect of imperfect elasticity in aneroid barometers appear to be three in number, viz.:

I. Select a specially good quality material for the diaphragm. The authors state that they have been able to find nothing superior to hard German silver. But it is important to reduce the thickness of the diaphragm, which they have carried with advantage from the usual .2 mm. down to .05 mm. The diaphragms should be pressed into shape by hand and thereby hardened, for softening was found to make the after-effect excessive.

II. Use a stiff spring so as to have large values of f. If this diminishes the sensitivity of the aneroid too much, one can restore this sensitivity somewhat by increasing the multiplying power. If this in turn entails too much friction in the mechanism, resort can be made to the vibration device of Mr. Goeppel. This consists of a steel plate attached to the aneroid and caused to vibrate by means of a toothed wheel operated by hand.

III. The foregoing theoretical equations assume vacuum boxes with only a single diaphragm. If both the top and bottom are made flexible, the equations can readily be modified; for example, 7b becomes

$$\frac{z}{p - p_o} = \frac{a}{f + \frac{1}{2}Q} \qquad (8)$$

The double surface box reduces the influence of after-effect, therefore, to about one-half what it would be otherwise.

These three principles have been applied to a number of aneroids constructed in the Reichsanstalt shops, the constants of which the authors proceed to give, and the effect of substituting various stiffnesses for the spring and making certain changes of multiplying power are reported numerically. The numerical effect of these substitutions on the observed hysteresis loops is finally given and shown to be quite marked.

In their concluding summary, the authors state that by means of the three methods given, they have succeeded in producing aneroids for which the greatest width of the hysteresis loop between 760 and 410 mm. is not over 2 mm. (It is not very clear how much time the authors allowed for "drift" to take place at the minimum pressure during the tests which established these hysteresis figures; it is merely stated that the rate of change of pressure was about 5 mm. per minute. Their result of slightly over one-half of one percent is exceptionally good, although by no means surprising if there were direct-return pressure cycles, that is, without prolonged delay at the lowest pressure.)